The Measurement of Elasticities

General meaning of elasticity of demand:

From the theory of demand we know that the amount of a commodity purchased per unit of time is a function of or depends on the price of the commodity, money incomes, the price of other (related) commodities, tastes, and the number of buyers of the commodity in the market. A change in any of the above factors will cause a change in the amount of the commodity purchased per unit of time. The elasticity of demand measures the relative responsiveness in the amount purchased per unit of time to a change in any one of the above factors, while keeping the others constant.

Price Elasticity Of Demand

The coefficient of *price elasticity of demand (e)* measures the percentage change in the quantity of a commodity demanded per unit of time resulting from a given percentage change in the price of the commodity. Since price and quantity are inversely related, the coefficient of price elasticity of demand is a negative number. In order to avoid dealing with negative values, a minus sign is often introduced into the formula for e. Letting ΔQ represent the change in the quantity demanded of a commodity resulting from a given change in its price (ΔP), we have

$$e = -\frac{\Delta Q/Q}{\Delta P/P} = -\frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$$

Demand is said to be *elastic* if $e > 1$, *inelastic* if $e < 1$ and *unitary elastic* if $e = 1$.

EXAMPLE 1. Given the market demand schedule in Table 1.1 and market demand curve in Fig. 1-1, we can find e for a movement from point *B* to point *D* and from *D* to *B*, as follows:

Table 1.1

Point	P_x($)	Q_x
A	8	0
B	7	1000
C	6	2000
D	5	3000
F	4	4000
G	3	5000
H	2	6000
L	1	7000
M	0	8000

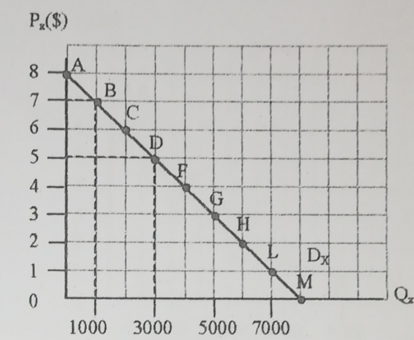

Fig.1-1

From B to D,

$$e=-\frac{Q_D-Q_B}{P_D-P_B}\cdot\frac{P_B}{Q_B}=-\left(\frac{2000}{-2}\right)\left(\frac{7}{1000}\right)=7$$

From D to B,

$$e=-\frac{Q_B-Q_D}{P_B-P_D}\cdot\frac{P_D}{Q_D}=-\left(\frac{-2000}{2}\right)\left(\frac{5}{3000}\right)\cong 1.67$$

We can avoid getting different results by using the average of the two prices $[(P_B+P_D)/2]$ and the average of the two quantities $[(Q_B+Q_D)/2]$ instead of either P_B and Q_B or P_D and Q_D in the formula to find e. Thus,

$$e=-\frac{\Delta Q}{\Delta P}\cdot\frac{(P_B+P_D)/2}{(Q_B+Q_D)/2}=-\frac{\Delta Q}{\Delta P}\cdot\frac{P_B+P_D}{Q_B+Q_D}$$

Applying this modified formula to find e either for a movement form B to D or for a movement from D to B we get

$$e=-\left(-\frac{2000}{2}\right)\left(\frac{12}{4000}\right)=3$$

This is the equivalent of finding e at the point midway between B and D (i.e., at point C).

Example-2. Given the market demand schedule in Table 1.2 and the market demand curve in Fig. 1-2, we can find e for a movement from point C to point F, from F to C and midway between C and F, as follows:

Table 1.2

Point	$P_x(\$)$	Q_x
A	7	500
B	6	750
C	5	1250
D	4	2000
F	3	3250
G	2	4750
H	1	8000

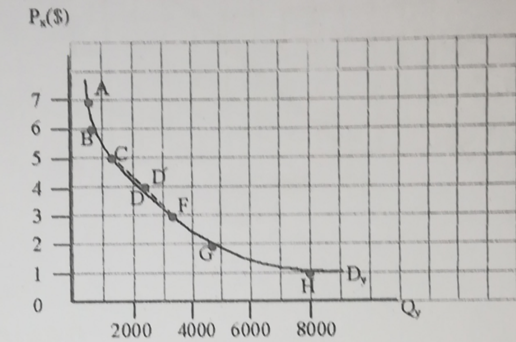

Fig.1-2

From C to F,

$$e=-\frac{\Delta Q}{\Delta P}\cdot\frac{P_C}{Q_C}=-\left(\frac{2000}{-2}\right)\left(\frac{5}{1250}\right)=4$$

From F to C,

$$e=-\frac{\Delta Q}{\Delta P}\cdot\frac{P_F}{Q_F}=-\left(\frac{-2000}{2}\right)\left(\frac{3}{3250}\right)\cong 0.92$$

At the point midway between C and F (point D′ on the dashed chord),

$$e=-\frac{\Delta Q}{\Delta P}\cdot\frac{(P_C+P_F)}{(Q_C+Q_F)}=-\left(-\frac{2000}{2}\right)\left(\frac{8}{4500}\right)\cong 1.78$$

POINT ELASTICITY

Point elasticity of demand: The coefficient of price elasticity of demand at a particular point on a demand curve.

EXAMPLE 3. We can find the elasticity of demand curve in Example 1 at point C geometrically as follows. (For easy reference, Fig. 1-1, with some modifications, is repeated here as Fig. 1-3). Since we want to measure elasticity at point C, we have only a single price and a single quantity. Expressing each of the values in the formula for e in terms of distances, we get:

$$e = -\frac{\Delta Q}{\Delta P}\cdot\frac{P}{Q}$$

$$= \frac{NM}{NC}\cdot\frac{NC}{ON}$$

$$= \frac{NM}{ON} = \frac{6000}{2000} = 3$$

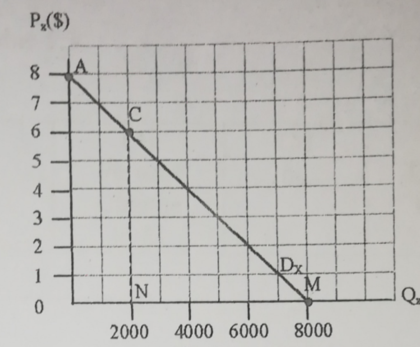

Fig.1-3

INCOME ELASTICITY OF DEMAND

The coefficient of income elasticity of demand (e_M) measures the percentage change in the amount of a commodity purchased per unit of time *($\Delta Q/Q$)* resulting from a given percentage change in a consumer's income *($\Delta M/M$)*. Thus

$$e_M = \frac{\Delta Q/Q}{\Delta M/M} = \frac{\Delta Q}{\Delta M}\cdot\frac{M}{Q}$$

When e_M is negative, the good is inferior. If e_M is positive, the good is normal. A normal good is usually a *luxury* if its $e_M > 1$, otherwise it is a *necessity*. Depending on the level of the consumer's income, e_M for a good is likely to vary considerably. Thus a good may be a luxury at "low" levels of income, a necessity at "intermediate" levels of income and an inferior good at "high" levels of income.

EXAMPLE 4. Columns (1) and (2) of Table 1.3 show the quantity of commodity X that an individual would purchase per year at various income levels. Column (5) gives the coefficient of income elasticity of demand of this individual for commodity *X between* the various successive levels of available income. Column

(6) indicates the range of income over which commodity X is a luxury, a necessity or an inferior good.

Table 1.3

(1) Income (M) ($/year)	(2) Quantity of X (units/year)	(3) Percent Change in Q_x	(4) Percent Change in M	(5) e_M	(6) Type of Good
8,000	5				
		100	50	2	luxury
12,000	10				
		50	33.33	1.50	luxury
16,000	15				
		20	25	0.80	necessity
20,000	18				
		11.11	20	0.56	necessity
24,000	20				
		-5	16.67	0.30	inferior
28,000	19				
		-5.26	14.29	0.37	inferior
32,000	18				

CROSS ELASTICITY OF DEMAND

The coefficient of *cross elasticity of demand* of commodity X with respect to commodity *Y*(e_{xy}) measures the percentage change in the amount of X purchased per unit of time ($\Delta Q_x/Q_x$) resulting from a given percentage change in the price of Y ($\Delta P_y/P_y$). Thus

$$e_{xy} = \frac{\Delta Q_X/Q_X}{\Delta P_y/P_y} = \frac{\Delta Q_X}{\Delta P_y} \cdot \frac{P_y}{Q_X}$$

If X and Y are substitutes, e_{xy} is positive. On the other hand, if X and Y are complements, e_{xy} is negative. When commodities are nonrelated (i.e., when they are independent of each other), $e_{xy} = 0$.

EXAMPLE 5. To find the cross elasticity of demand between tea (X) and coffee (Y) and between tea (X) and lemons (Z) for the data in the next table, we proceed as follows.

Table 1.4 (a)

Commodity	Before		After	
	Price (cents/cup)	Quantity (units/month)	Price (cents/unit)	Quantity (units/month)
Coffee (Y)	40	50	60	30
Tea (X)	20	40	20	50

Table 1.4 (b)

Commodity	Before		After	
	Price (cents/unit)	Quantity (units/month)	Price (cents/unit)	Quantity (units/month)
Lemon (Z)	10	20	20	15
Tea (X)	20	40	20	35

$$e_{xy} = \frac{\Delta Q_x}{\Delta P_y} \cdot \frac{P_y}{Q_x} = \left(\frac{+10}{+20}\right)\left(\frac{40}{40}\right) = +0.5$$

$$e_{xz} = \frac{\Delta Q_x}{\Delta P_z} \cdot \frac{P_z}{Q_x} = \left(\frac{-5}{+10}\right)\left(\frac{10}{40}\right) = -0.125$$

Since e_{xy} is positive, tea and coffee are substitutes. Since e_{xz} is negative, tea and lemons are complements.